Key

3.5/6 Solving Systems of Three Linear Equations in Three Variables
The Elimination Method and matrices

The solution to a system of three linear equations in three variables is an ordered triple. (x, y, z)

Example 1. The solution must be a solution of all 3 equations.
Is (-3, 2, 4) a solution of this system?

$3x + 2y + 4z = 11$ $\quad 3(-3) + 2(2) + 4(4) = 11 \quad -9 + 4 + 16 = 11$ ✓
$2x - y + 3z = 4$ $\quad 2(-3) - (2) + 3(4) = 4 \quad -6 - 2 + 12 = 4$ ✓
$5x - 3y + 5z = -1$ $\quad 5(-3) - 3(2) + 5(4) = -1 \quad -15 - 6 + 20 = -1$ ✓

Example 2
Use elimination to solve the following system of equations.

① $x - 3y + 6z = 21$
② $3x + 2y - 5z = -30$
③ $2x - 5y + 2z = -6$

Step 1. Rewrite the system as two smaller systems, each containing two of the three equations.

① $x - 3y + 6z = 21$
② $3x + 2y - 5z = -30$

mult #1 by -3: $-3x + 9y - 18z = -63$
$3x + 2y - 5z = -30$
$11y - 23z = -93$

① $x - 3y + 6z = 21$ mult. #1 by -2
③ $2x - 5y + 2z = -6$
$-2x + 6y - 12z = -42$
$2x - 5y + 2z = -6$
$y - 10z = -48$

combine to eliminate x

combine to eliminate y

$11y - 23z = -93$
$y - 10z = -48$ mult. by -11 →
$11y + 110z = +528$
$-11y - 23z = -93$
$87z = 435$
$z = 5$

plug in

$y - 10(5) = -48$
$y = 2$

sub in y

$x - 3(2) + 6(5) = 21$
$x = -3$

sub in 2+y into ①

$(-3, 2, 5)$

Step 2. Eliminate THE SAME variable in each of the two smaller systems. Any variable will work, but sometimes one may be a bit easier to eliminate.
Step 3 Write the resulting equations in two variables together as a system of equations. Solve the system for the two remaining variables.
Step 4 Substitute the value of the variables from the system of two equations in one of the ORIGINAL equations with three variables.

choose a variable that looks easy to eliminate!

Step 5. Check solution in all three equations.

all mult of 5 so eliminate x first

Due in class. Practice: ① $-5x + 3y + z = -15$

② $10x + 2y + 8z = 18$

③ $15x + 5y + 7z = 9$

① ② $-5x + 3y + z = -15$

mult by (2) $10x + 2y + 8z = 18$

$-10x + 6y + 2z = -30$

$8y + 10z = -12$

① ③ $-5x + 3y + z = -15$

mult by (3) $15x + 5y + 7z = 9$

$-15x + 9y + 3z = -45$

$14y + 10z = -36$

$8y + 10z = -12$

$14y + 10z = -36$

$\times -1$ $-8y - 10z = 12$

$6y = -24$

$y = -4$

$8(-4) + 10z = -12$

$10z = 20$

$z = 2$

$-5x + 3(-4) + 2 = -15$

-12

$-5x = -5$

$x = 1$

$(1, -4, 2)$

Example 3. The operator of a ski resort sells three types of tickets: full-day skiing at $40, half-day skiing at $25, and rental of ski equipment at $15. At the end of the day, he finds that he has sold a total of 517 tickets. The total number of people skiing during the day was counted at 425. He also has a total cash intake of $16 505.
Determine the sales of each type of ticket.

x = full day ticket #
y = half day ticket #
z = rental #

total $x + y + z = 517$

skiers $x + y = 425$

money $40x + 25y + 15z = 16505$

(300, 125, 92)

eliminate z from total & money

$-15x - 15y - 15z = -7755$
$40x + 25y + 15z = 16505$
$25x + 10y = 8750$

$25x + 10y = 8750$
$-10x - 10y = -4250$
$15x = 4500$
$x = 300$

$300 + y = 425$
$y = 125$

$300 + 125 + z = 517$
$z = 92$

RREF calculator notes. Example 4.
Write as a matrix:

$$\begin{cases} 4x + y - 2z = 3 \\ 2y + z = 4 \\ 3x - 3y - z = 9 \end{cases} \qquad \left[\begin{array}{ccc|c} 4 & 1 & -2 & 3 \\ 0 & 2 & 1 & 4 \\ 3 & -3 & -1 & 9 \end{array}\right]$$

$$\begin{cases} 5x - 2y + z = -1 \\ -x - y - 2z = 5 \\ 3x + 2y + 2z = 2 \end{cases} \qquad \left[\begin{array}{ccc|c} 5 & -2 & 1 & -1 \\ -1 & -1 & -2 & 5 \\ 3 & 2 & 2 & 2 \end{array}\right]$$

$$\begin{cases} 3x + 5z = -4 \\ -2x + y - 3z = 9 \\ -x - 2y + 9z = 0 \end{cases} \qquad \left[\begin{array}{ccc|c} 3 & 0 & 5 & -4 \\ -2 & 1 & -3 & 9 \\ -1 & -2 & 9 & 0 \end{array}\right]$$

You must align the variables (use 0 if the variable is missing)

Keys to press.
2nd matrix: edit menu. For system with 3 equations the size is 3 x 4

1. Write the system as a matrix.
2. Enter the matrix into calculator. (Matrix edit)
3. Use RREF key under the matrix MATH tab.
4. Recall your matrix using the NAMES tab. Hit enter.

Example 5. A company sells three types of movie gift baskets. A basic basket with 2 movie passes and 1 package of microwave popcorn costs $15.50. A medium basket with 2 movie passes, 2 packages of popcorn, and 1 DVD costs $37. A super basket with 4 movie passes, 3 packages of popcorn, and 2 DVDs costs $72.50. Find the cost of each item in the gift baskets.

m = movie pass # p = popcorn # D = DVD #

Equations

$$2M + P = 15.50$$
$$2M + 2P + D = 37$$
$$4M + 3P + 2D = 72.50$$

Matrix:

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 15.50 \\ 2 & 2 & 1 & 37.00 \\ 4 & 3 & 2 & 72.50 \end{array}\right]$$

$M = 7$
$P = 1.50$
$D = 20$

Solution:

PRACTICE DUE IN CLASS.

There are 49,000 seats in a sports stadium. Tickets for the seats in the upper level sell for $25, the ones in the middle level cost $30, and the ones in the bottom level are $35 each. The number of seats in the middle and bottom levels together equals the number of seats in the upper level. When all of the seats are sold for an event, the total revenue is $1,419,500. How many seats are there in each level? U = Upper M = Middle B = Bottom

Equations:

$$U + M + B = 49000$$

rewrite → $M + B = U \rightarrow -U + M + B = 0$

$$25U + 30M + 35B = 1419500$$

Matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 49000 \\ -1 & 1 & 1 & 0 \\ 25 & 30 & 35 & 1419500 \end{array}\right]$$

Solution:

$U = 24500$
$M = 10,100$
$B = 14,400$